الفريق الاكاديمي اسألني عن الهندسة

يقدم لكم

ملخص

calculus 102

مادة المد

Subject Date No.

6.2 Volumes ...

+

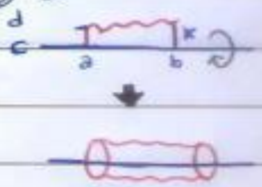
6.3 Volumes by Cylindrical Shells

↳ Note :- if rotation is about or parallel to [x-axis] integration is done in relation to [dx].

↳ if rotation is about or parallel to [y-axis] integration is done in relation to [dy].

↳ Methods of finding volume :-

ⓐ Disk method

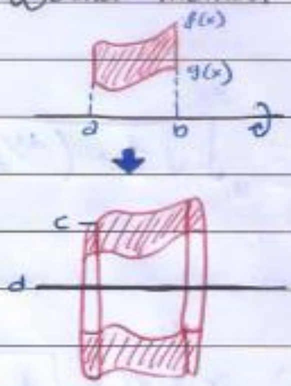


$$V = \pi \int_a^b [f(x)]^2 \cdot dx$$

or

$$\pi \int_d^c [f(y)]^2 \cdot dy$$

ⓑ Washer method



$$V = \pi \int_a^b [f(x)]^2 - [g(x)]^2 \cdot dx$$

or

$$\pi \int_d^c [f(y)]^2 - [g(y)]^2 \cdot dy$$

N O T E B O O K

Subject Date No.

© Cylindrical Shells

↳ when rotation of given shape only gives a hollow object (shell)

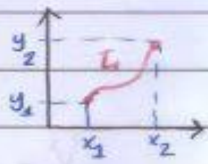
$$V = 2\pi \int_d^c r h \, dy \quad \text{or} \quad 2\pi \int_b^a r h \, dx$$

↳ when dy → h is taken parallel to y
when dx → h is taken parallel to x

↳ radius → distance from axis of rotation → if two equations :- (big - small)
height → parallel to axis of rotation

8.1 Arc Length
+
8.2 Area of a Surface of Revolution

↳ Arc length



$$L = \int_{x_1}^{x_2} \sqrt{1 + (dy/dx)^2} \, . \, dx \quad \boxed{\text{or}} \quad \int_{y_1}^{y_2} \sqrt{1 + (dx/dy)^2} \, . \, dy$$

↳ Area of surface of revolution (when a curve is rotated about axis)

$$A = 2\pi \int_{x_1}^{x_2} f(x) \sqrt{1 + (dy/dx)^2} \, . \, dx \quad \boxed{\text{or}} \quad 2\pi \int_{y_1}^{y_2} f(x) \sqrt{1 + (dx/dy)^2} \, . \, dy$$

NOTEBOOK

Subject Date No.

Ch.11 Infinite Sequences & Series

↳ Sequence :- a function whose domain is a subset of non-negative integers.

* a sequence $\{a_n\}$ has a limit (L), where

$$\lim_{n\to\infty} (a_n) = L$$

if limit exists ⇒ converge, otherwise ⇒ diverge

* all limit laws & the squeezing theorem can be adapted to sequences

* Limit laws for sequences :-

1) $\lim_{n\to\infty} (a_n \pm b_n) = \lim_{n\to\infty} (a_n) \pm \lim_{n\to\infty} (b_n)$
2) $\lim_{n\to\infty} c(a_n) = [\lim_{n\to\infty} a_n] c$ (where c is a constant)
3) $\lim_{n\to\infty} (a_n b_n) = (\lim_{n\to\infty} a_n)(\lim_{n\to\infty} b_n)$
4) $\lim_{n\to\infty} (a_n / b_n) = (\lim_{n\to\infty} a_n) / (\lim_{n\to\infty} b_n)$ $(b_n \neq 0)$
5) $\lim_{n\to\infty} (a_n)^p = (\lim_{n\to\infty} a_n)^p$
6) $\lim_{n\to\infty} |a_n| = 0$ then $\lim_{n\to\infty} a_n = 0$
7) L'Hopital rule
8) $\lim_{n\to\infty} a_n = L$ then $\lim_{n\to\infty} (a_{n+1}) = L$

Subject Date No.

* Monotonic Sequence
  - a sequence is monotonic if it's either increasing or decreasing :
    - increasing $a_n < a_{n+1}$
    - decreasing $a_n > a_{n+1}$

* All sequences are bounded
  - Bounded above $a_n \leq M$
  - Bounded below $m \leq a_n$

* All bounded, monotonic sequences are converge.

- Series :- a sum of a sequence

  * notation :- $\sum(a_n) = a_0 + a_1 + a_2 + \dots + a_n = S_n$

  * $\lim_{n\to\infty} S_n = \sum_{k=1}^{\infty} a_k$
    - exists ⇒ converge
    - does NOT exist ⇒ diverge

  * Geometric Series
    - $\sum_{n=1}^{\infty} ar^n$ = geometric series
    - $r = a_{n+1}/a_n$ | $|r| < 1$ ⇒ conv. ; $|r| \geq 1$ ⇒ div.
    - $S_n = a_1/(1-r)$

Subject Date No.

↳ Converge / Diverge Tests :-

① Divergence Test

if $\lim_{n\to\infty} a_n \neq 0$ or does not exist

then $\sum a_n \Rightarrow$ diverge.

② Convergence Test

if $\lim_{n\to\infty} a_n = 0$ then $\sum a_n \Rightarrow$ converge

③ Integral Test

if $\int_1^\infty f(x)\,dx$ is converge then $\sum a_n \Rightarrow$ converge

if $\int_1^\infty f(x)\,dx$ is diverge then $\sum a_n \Rightarrow$ diverge

where $f(x) = a_n$ and $f(x)$ is a +ve, continuous, decreasing function on $[1, \infty)$.

④ P-series

$$\sum_{n=1}^{\infty} 1/n^p \begin{cases} p>1 \Rightarrow \text{converge} \\ p\leq 1 \Rightarrow \text{diverge} \end{cases}$$

Subject Date No.

(5) Comparsion Test

($\sum a_n$ & $\sum b_n$ are series with +ve terms)

if $\sum b_n$ is converge & $a_n \le b_n$, $\sum a_n \Rightarrow$ converge

if $\sum b_n$ is diverge & $a_n \ge b_n$, $\sum a_n \Rightarrow$ diverge

(6) Limit Comparsion Test

if $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = C$, $C > 0$ both a_n & b_n are converge or diverge.

(7) Alternating Series Test

alternating series :- $\sum_{n=1}^{\infty} (-1)^{n-1} a^n$

if $\begin{cases} a_{n+1} \le a_n \text{ (decreasing series)} \\ \lim_{n\to\infty} a_n = 0 \end{cases}$

then, $\sum_{n=1}^{\infty} a_n$ is converge

(8) Absolutely Converge

if $\sum |a_n|$ is converge, then $\sum a_n \Rightarrow$ absolutely converge

N O T E B O O K

Subject　　　　Date　　　　No.

(9) Conditionally Converge

if $\sum a_n$ is converge and $\sum |a_n|$ is diverge $\Rightarrow$ then $\sum a_n$ is conditionally converge

(10) Ratio Test

(i) if $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L < 1 \Rightarrow \sum_{n=1}^{\infty} a_n$ is absolutely converge

(ii) if $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or $L = \infty \Rightarrow \sum_{n=1}^{\infty} a_n$ is diverge

(iii) if $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1 \Rightarrow$ no conclusion can be made (test fails)

(11) Root Test

(i) if $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1 \Rightarrow \sum_{n=1}^{\infty} a_n$ is absolutely converge

(ii) if $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1$ or $L = \infty \Rightarrow \sum_{n=1}^{\infty} a_n$ is diverge

(iii) if $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1 \Rightarrow$ no result (test fails)